Programme and Abstracts

XXVIII Incontro AILA

Udine, Italy
September 03-06, 2024

Contents

Preface

The scientific programme of XXVIII Incontro Aila consists of:

- talks by 10 invited speakers, namely:
  - Claudio Agostini (Technische Universität Wien, Vienna)
  - Ivano Ciardelli (Università di Padova, Padova)
  - Thomas Ehrhard (Université Paris Cité, Paris)
  - Marta Fiori Carones (Sobolev Institute of Mathematics, Novosibirsk)
  - Francesco Gallinaro (Università di Pisa, Pisa)
  - Mai Gehrke (Université Côte d'Azur, Nice)
  - Martino Lupini (Università di Bologna, Bologna)
  - Elvira Mayordomo (Universidad de Zaragoza, Zaragoza and Iowa State University, Ames)
  - Sandra Müller (Technische Universität Wien, Vienna)
  - Alessandro Vignati (Université Paris Cité, Paris)
- several contributed talks.

The event is made possible thanks to the financial support of:

- Università degli Studi di Udine
- Dipartimento di Scienze Matematiche, Informatiche e Fisiche
- Istituto Nazionale di Alta Matematica - GNSAGA
- Associazione Italiana di Logica e sue Applicazioni

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	Tuesday	Wednesday
9.00-10.00	Registration	Mayordomo
10.00-10.30	Coffee break	Coffee break
10.30-11.30	Ehrhard	Gehrke
11.40-12.40	Contributed talks	Contributed talks
12.40-14.30	Lunch break	Lunch break
14.30-16.10	Contributed talks	Contributed talks
16.10-16.40	Coffee break	Coffee break
16.40-17.40	Lupini	Vignati

	Thursday	Friday
9.00-10.00	Agostini	Fiori Carones
10.00-10.30	Coffee break	Coffee break
10.30-11.30	Müller	Ciardelli
11.40-12.40	Contributed talks	Gallinaro
12.40-14.30	Lunch break	Lunch break
14.30-16.10	Contributed talks	–
16.10-16.40	Coffee break	–
16.40-18.40	AILA members' meeting	–

Invited Talks

Thomas Ehrhard

Differential linear logic: from semantics to syntax

In 1969, Dana Scott met Christopher Strachey in Oxford. Together they created a new branch of fundamental computer science: *denotational semantics*. Since the mid 60's, Strachey was advocating a *mathematical semantics* of programs independent from their implementation on actual computers. Due to the increasing number of computer architectures and of programming languages, the need of such a semantics became indeed more and more critical. Denotational semantics is based on new ideas of Scott which allowed him in 1968 to build the complete lattice D_∞, solving a long-standing problem: find a mathematical interpretation of pure lambda-terms allowing to see them as the functions they are intuitively denoting.

Denotational semantics was mainly considered of a topological nature until 1986, when Jean-Yives Girard discovered *linear logic*, recasting proofs and programs in a setting much closer to linear algebra (often in infinite dimension, where topology may become necessary). The denotational models of linear logic are intuitively categories of linear morphisms equipped with an exponential, a modality (technically, a comonad) allowing them to host also non-linear morphisms which are intuitively to be considered as analytic functions. One major feature of linear logic is to deeply relate algebraic linearity with operational linearity: a program is linear when it uses completely and exactly once its argument.

Linear logic has a pivotal rule called *dereliction* allowing to see a linear morphism as an analytic one, simply forgetting linearity. In most models of linear logic, this rule has a kind of inverse which can be understood as an operation of "differentiation at 0". In the early 2000's I observed that, in several interesting models of linear logic, this *codereliction* rule is complemented with additional rules, symmetric to the usual structural rules associated in linear logic with the exponential. Altogether, these new rules allow to compute the differential of an arbitrary proof of linear logic. Starting from this observation, with Laurent Regnier, we introduced *differential linear logic* and the *differential lambda-calculus*.

In this talk, I will present this global conceptual framework and explain how we developed the differential lambda-calculus, introducing in particular the Taylor expansion of lambda-terms which uses iterated differentiation of terms and provides a fine-grain algebraic theory of program approximations. I will also discuss the need to equip the differential lambda-calculus with an addition operation on terms, the operational and denotational consequences of this operation, and how it can be controlled in the recently discovered setting of *coherent differentiation*.

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Martino Lupini

Borel-definable algebraic topology

Recently, Bergfalk, Panagiotopoulos, and I have introduced refinements of classical algebraic invariants endowed with additional information of descriptive set-theoretic nature. I will present an overview of applications of such Borel-definable invariants to algebra and topology, obtained jointly with Bergfalk, Casarosa, Codenotti, Meadows, Sarti, and Panagiotopoulos.

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Elvira Mayordomo

On point to set principles, normality, and algorithmic randomness

Effective and resource-bounded dimensions were defined by Lutz in [6] and [5] and have proven to be useful and meaningful for quantitative analysis in the contexts of algorithmic randomness, computational complexity and fractal geometry (see the surveys [2, 7, 3, 10] and all the references in them).

The point-to-set principle of J. Lutz and N. Lutz [8] fully characterizes Hausdorff and packing dimensions in terms of effective dimensions in the Euclidean space, enabling effective dimensions to be used to answer open questions about fractal geometry, with already an interesting list of geometric measure theory results (see [4, 9]).

Finite state dimension [1] is the lowest level effectivization of Hausdorff dimension and is closely related to Borel normality. In this talk I will review its main properties, prove a new characterization in terms of information content approximated at a certain precision, and consider new generalizations of normality. I will then prove a finite-state dimension point to set principle [11].

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Acknowledgements: Research supported in part by Spanish Ministry of Science and Innovation grant PID2022-138703OB-I00 and by the Science dept. of Aragon Government: Group Reference T64_20R (COSMOS research group).

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Mai Gehrke

Stone duality for spectral sheaves and the patch monad

Sheaf representations may be viewed as generalising Stone's representation theorem and the ensuing duality for Boolean algebras. In this setting, we establish a duality between global sheaves on spectral spaces and a category of very simple idempotent semigroups known as right distributive bands. This is a sheaf-theoretical extension of classical Stone duality between spectral spaces and bounded distributive lattices. The topology of a spectral space admits a refinement, the so-called patch topology, giving rise to a monad on sheaves over a fixed spectral space which we call the patch monad. Under the duality just mentioned the algebras of this patch monad are shown to correspond to distributive skew lattices, where skew lattices are a non-commutative variant of lattices originating in quantum logic and operator algebra. The research reported on in this talk is joint work with Clemens Berger.

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Alessandro Vignati

Rigidity, Rigidity, Rigidity!

We are interested in the effects of additional set theoretic axioms on quotient structures and their isomorphisms. Our focus is on rigidity, measured in terms of existence (or rather non-existence) of suitably non-trivial isomorphisms of the quotients in question. Consider for example the Boolean algebra $\mathcal{P}(\omega)/\mathrm{Fin}$: Forcing axioms imply that all of its automorphisms are trivial, while under the Continuum Hypothesis this rigidity fails. This behavior is the template around which this area of work revolves, and in this talk we consider some of its generalizations. We present a variety of situations where analogous patterns persist, such as (reduced products of) Boolean algebras, graphs, groups, or linear orders, but also more analytic objects such as Čech–Stone remainders, C^*-algebras or even objects constructed from coarse geometrical data.

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Claudio Agostini

Classes of "big" spaces from characterizations of metrizability

Metrizable spaces are a key concept in numerous areas of mathematics, including descriptive set theory. Consequently, many different characterizations of metrizability have been proposed.

Each characterization brings light on the crucial topological properties that guarantee the desirable behavior of metric spaces. These properties are particularly valuable in generalized descriptive set theory, where they help define new classes of non-first-countable (and thus non-metrizable) topological spaces. This enables the extension of classical descriptive set theory results pertaining to Polish spaces to these new classes of spaces.

Various theorems offer different properties that define distinct classes of spaces, leading to questions about the existence of a preferable class and the sufficiency of specific properties for deriving certain theorems.

In this talk, I will examine different characterizations of metrizability, compare them, and show how some results from classical descriptive set theory can be recovered from certain properties and not from others.

This is joint work with Luca Motto Ros.

Acknowledgements: Supported by the FWF grant P35655-N

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Sandra Müller

Infinite Games and Large Cardinals: Attacking Independence by Connecting the Hierarchies

The independence phenomenon is a central theme in set theory. Moreover, starting from the Continuum Problem, there are numerous statements in various areas of mathematics that can neither be proven nor disproven in the usual axiomatic framework given by the Zermelo Fraenkel Axioms. The most promising approach to attack this issue is by studying extensions of the usual axiomatic framework, their connections, and their impact on previously independent statements. Two famous examples in set theory are the hierarchies given by the determinacy of infinite two-player games and large cardinals. The deep connection between these two hierarchies forms the backbone of inner model theory. We outline this connection and discuss recent progress as well as important open questions in the area.

Acknowledgements: This research was funded in whole or in part by the Austrian Science Fund (FWF) [10.55776/V844, 10.55776/Y1498, 10.55776/I6087]. For open access purposes, the author has applied a CC BY public copyright license to any author accepted manuscript version arising from this submission.

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Marta Fiori Carones

The strength of a theorem of Rival and Sands

(Joint work with Paul Shafer and Giovanni Soldà)

In 1979 Ivan Rival and Bill Sands proved that each infinite graph G has an infinite subgraph H such that each vertex of G is adjacent to none or to one or to infinitely many vertices of H. This statement, showing the existence of a substructure with some property in every infinite graphs, resembles Ramsey's theorem for pairs, which guarantees the existence of a complete or a totally disconnected subgraph in each infinite undirected graph.

We investigated the strength of this statement (restricted to countable graphs) from the viewpoint of Weihrauch reducibility and reverse mathematics. During the talk variants of this theorem and their strength are also presented.

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Ivano Ciardelli

Inquisitive neighborhood logic

In this talk, I will first introduce the motivation and the key ideas of the ongoing research program on inquisitive modal logic. I will then present a particular system of inquisitive modal logic designed to talk about properties of neighborhood structures, whose basic modality is a sort of strict conditional quantifying over neighborhoods. I will discuss two concrete interpretations of this logic, characterize its expressive power in terms of a natural notion of bisimulation, and present complete axiomatizations for some salient classes of frames.

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Francesco Gallinaro

Model theory of complex analytic functions: quasiminimality and existential closedness

A (model-theoretic) structure S is *quasiminimal* if all the definable subsets of S are countable or have countable complement. In the 1990s, Zilber made a striking conjecture [3] predicting that the complex numbers seen as a structure in the language of rings expanded by a symbol for the complex exponential function should be quasiminimal.

This conjecture soon turned out to be related to several results and open problems from transcendental number theory and Diophantine geometry, and in turn opened up new lines of research, both in logic and geometry. In logic, it led to the development of a theory for quasiminimal structures, with categoricity theorems for classes of quasiminimal structures axiomatizable in some infinitary logic. In geometry, *existential closedness* problems were introduced: these are concerned with the existence of solutions to systems of equations involving polynomials and certain analytic functions, such as the complex exponential or the modular j invariant. Bays and Kirby's result from [1] was an important milestone, establishing that the quasiminimality conjecture for exp (a purely model-theoretic statement) would follow from the *exponential-algebraic closedness conjecture*, which predicts the existence of solutions of appropriate systems of polynomials and exponentials.

In this talk I will survey some of these conjectures and the relations between them, and some partial results towards them. In particular, I will discuss the result of Kirby and myself [2] about quasiminimality of the complex field expanded with multivalued *power functions*, where for each $\lambda \in \mathbb{C}$ the power function $w \mapsto w^\lambda$ on $\mathbb{C}^\times$ maps $w \in \mathbb{C}^\times$ to the set of determinations of $\exp(\lambda \log w)$.

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Contributed Talks

TBA

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